EXERCISE – III SUBJECTIVE QUESTIONS

1. (a) Find the equation of the ellipse with its centre (1, 2), focus at (6, 2) and passing through the point (4, 6).

Sol.

(b) An ellipse passes through the points (–3, 1) & (2, –2) & its principal axis are along the coordinate axes in order. Find its equation.

Sol.

2. The tangent at any point P of a circle $x^2 + y^2 = a^2$ meets the tangent at a fixed point A(a, 0) in T and T is joined to B, the other end of the diameter through A, prove that the locus of the intersection of AP and BT is an ellipse whose eccentricity is $1/\sqrt{2}$.

Sol.

3. The tangent at the point α on a standard ellipse meets the auxiliary circle in two points which subtends a right angle at the centre. Show that the eccentricity of the ellipse is $(1 + \sin^2\alpha)^{-1/2}$.

Sol.

4. If any two chords be drawn through two points on the major axis of an ellipse equidistant from the centre, show that $\tan\frac{\alpha}{2}.\tan\frac{\beta}{2}.\tan\frac{\gamma}{2}.\tan\frac{\delta}{2} = 1$, where $\alpha, \beta, \gamma, \delta$ are the eccentric angles of the extremities of the chords.

Sol.

5. If the normal at the point P(θ) to the ellipse $\frac{x^2}{14} + \frac{y^2}{5} = 1$, intersects it again at the point Q(2θ), show that $\cos\theta = -(2/3)$.

Sol.

6. If the normals at the points P, Q, R with eccentric angles α, β, γ on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ are concurrent, then show that, $\begin{vmatrix} \sin\alpha & \cos\alpha & \sin 2\alpha \\ \sin\beta & \cos\beta & \sin 2\beta \\ \sin\gamma & \cos\gamma & \sin 2\gamma \end{vmatrix} = 0$.

Sol.

7. Prove that the equation to the circle, having double contact with the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (with eccentricity e) at the ends of a latus rectum, is $x^2 + y^2 - 2ae^3x = a^2(1 - e^2 - e^4)$.

Sol.

8. Find the equations of the lines with equal intercepts on the axes & which touch the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$.

Sol.

9. Suppose x and y are real numbers and that $x^2 + 9y^2 - 4x + 6y + 4 = 0$ then find the maximum value of $(4x - 9y)$.

Sol.

10. A tangent having slope $-\frac{4}{3}$ to the ellipse $\frac{x^2}{18} + \frac{y^2}{32} = 1$, intersects the axis of x & y in points A & B respectively. If O is the origin, find the area of triangle OAB.

Sol.

11. 'O' is the origin & also the centre of two concentric circles having radii of the inner & the outer circle as 'a' & 'b' respectively. A line OPQ is drawn to cut the inner circle in P & the outer circle in Q. PR is drawn parallel to the y-axis & QR is drawn parallel to the x-axis. Prove that the locus of R is an ellipse touching the two circles. If the focii of this ellipse lie on the inner circle, find the ratio of inner : outer radii & find also the eccentricity of the ellipse.

Sol.

12. ABC is an isosceles triangle with its base BC twice its altitude. A point P moves within the triangle such that the square of its distance from BC is half the rectangle contained by its distance from the two sides. Show that the locus of P is an ellipse with eccentricity $\sqrt{2/3}$ passing through B & C.

Sol.

13. Let d be the perpendicular distance from the centre of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ to the tangent drawn at a point P on the ellipse. If F_1 & F_2 are the two foci of the ellipse, then show that

$(PF_1 - PF_2)^2 = 4a^2\left[1 - \frac{b^2}{d^2}\right]$.

Sol.

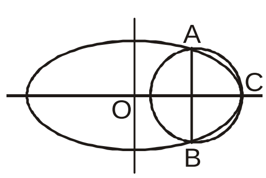
14. Common tangents are drawn to the parabola $y^2 = 4x$ & the ellipse $3x^2 + 8y^2 = 48$ touching the parabola at A & B and the ellipse at C & D. Find the area of the quadrilateral.

Sol.

15. If the normal at a point P on the ellipse of semi axes a, b & centre C cuts the major & minor axes at G & g, show that $a^2.(CG)^2 + b^2.(Cg)^2 = (a^2 - b^2)^2$. Also prove that $CG = e^2CN$, where PN is the ordinate of P.

Sol.

16. A circle intersects an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ precisely at three points A, B, C as shown in the figure. AB is a diameter of the circle and is perpendicular to the major axis of the ellipse. If the eccentricity of the ellipse is 4/5, find the length of the diameter AB is terms of a.



Sol.

17. The tangent at a point P on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ intersects the major axis in T & N is the foot of the perpendicular from P to the same axis. Show that the circle on NT as diameter intersects the auxiliary circle orthogonally.

Sol.

18. The tangents from (x_1, y_1) to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ intersect at right angles. Show that the normals at the points of contact meet on the line $\frac{y}{y_1} = \frac{x}{x_1}$.

Sol.

19. If the tangent at any point of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ makes an angle α with the major axis and an angle β with the focal radius of the point of contact then show that the eccentricity 'e' of the ellipse is given by the absolute value of $\frac{\cos\beta}{\cos\alpha}$.

Sol.

20. An ellipse has foci at $F_1(9, 20)$ and $F_2(49, 55)$ in the xy-plane and is tangent to the x-axis. Find the length of its major axis.

Sol.